# UNIT-I

## INTRODUCTION TO QUANTITATIVE ANALYSIS & DECISION THEORY

**History and development of Quantitative approach, Definition of Operations Research, quantitate analysis and decision making, Quantitative analysis Vs Qualitative Analysis.**

**Steps of Decision-Making Process, Types of Decision-Making environments: Under Uncertainty, Under Risk; Decision Tree Analysis.**

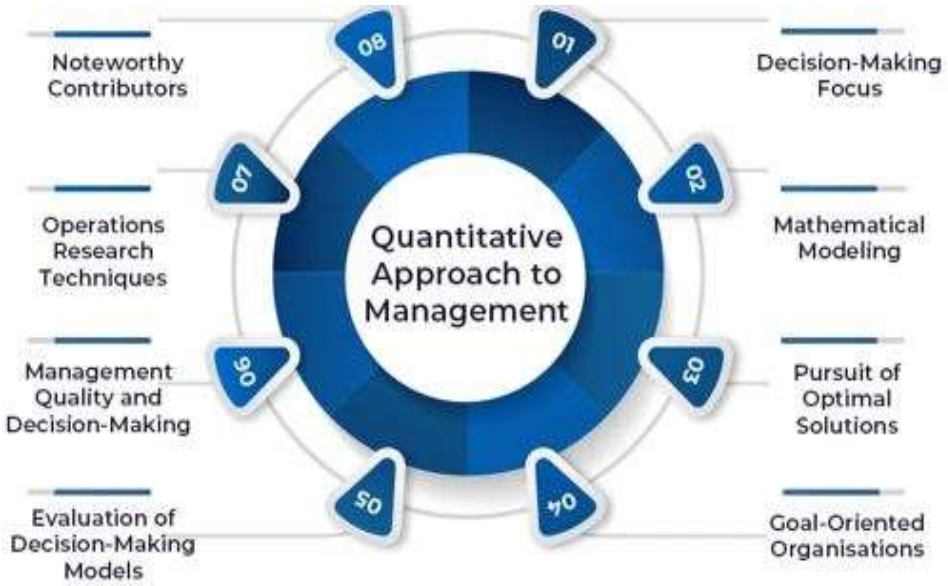
***Course Outcome***: *Apply the quantitative approach for effective decision-making.*

## Chapter Outlines: -

- **History and Development of Quantitative Approach**
- **Definitions of Operations Research**
- **Quantitative Vs Qualitative analysis**
- **Decision Making Process**
- **Types of Decision-making Environments**
- **Decision Tree Analysis**

## Quantitative Approach to Management

The quantitative approach, also referred to as the management science approach, arose in prominence during and following World War II. Its origins can be traced back to the pressing need to make informed decisions concerning strategic and tactical military operations. In response to this demand, multidisciplinary groups of scientists were assembled to conduct applied scientific research in this field.




The primary objective of these research groups was to devise optimal solutions for the deployment of military resources. They sought to tackle complex military and logistical problems by applying mathematical modelling techniques. These models aimed to represent and analyse the intricacies (complexities or difficulties) of the challenges at hand.

By utilizing mathematical models, researchers were able to simulate various scenarios, evaluate different options, and identify optimal solutions based on specific objectives and constraints. Techniques like optimization, simulation, and statistical analysis were employed to gain valuable insights and support the decision-making process. Following the war, the quantitative approach transcended its military origins and found applications in diverse sectors, including business, industry, healthcare, transportation, finance, and government. It emerged as an invaluable tool for addressing complex problems and enhancing decision-making processes by providing a systematic and data-driven framework.

## Features of the Quantitative Approach to Management

The following are the features of the quantitative approach:

1. **Decision-Making Focus:** At its core, the quantitative approach recognizes that management involves a series of decision-making processes. Managers are tasked with analysing interrelated variables and ultimately selecting the optimal solution.

2. **Mathematical Modelling:** A fundamental aspect of the quantitative approach is the utilization of mathematical models to represent decision situations. These models consist of functional equations that establish the quantitative relationships between variables.

3. **Pursuit of Optimal Solutions:** With proper formulation and accurate solution techniques, the quantitative approach aims to identify the best possible solution within the mathematical model. This pursuit of optimality enables decision-makers to make well-informed choices.

4. **Goal-Oriented Organisations:** The quantitative approach acknowledges that organisations exist to achieve specific and measurable economic objectives. To reach these goals, scientific reasoning supported by quantification is employed to make optimal decisions.

5. **Evaluation of Decision-Making Models:** It is important to assess decision-making models against predetermined criteria. By evaluating models, decision-makers can ensure their effectiveness and suitability in guiding the decision-making process.

6. **Management Quality and Decision-Making:** The quality of management is evaluated based on the quality of decisions made across diverse situations. By emphasizing informed and optimal decision-making, the quantitative approach seeks to enhance overall managerial effectiveness.

7. **Operations Research Techniques:** A wide range of operations research techniques is employed within the quantitative approach. These include linear programming, queuing theory, game theory, and probability theory. Such techniques have been developed and are extensively applied in both business and non-business domains.

8. ***Noteworthy Contributors:*** *Several influential thinkers have made significant contributions to the development of the quantitative approach. Among them are Herbert Simon, Von Newman, R.M. Cyert, James March, W.C. Churchman, and Russel Ackoff. Their contributions have shaped and advanced the field.*

# Uses and Limitations of Quantitative Approach to Management

The quantitative approach offers both benefits and limitations in managerial decision-making:

## Uses or advantages of the Quantitative Approach to Management:

- **Improved Decision-Making:** One of the key advantages of the quantitative approach is its emphasis on studying diverse decision situations and seeking ways to enhance them. By prioritizing factual data and logical analysis over intuition and hunches, this approach facilitates more precise and informed managerial decisions.
- **Encouragement of Disciplined Thinking:** The quantitative approach promotes disciplined thinking by advocating for the use of data and logical reasoning in decision-making processes. It highlights the importance of a systematic and analytical approach to enhance the overall quality of decisions.
- Allows for a broader study, involving a greater number of subjects, and enhancing the generalization of the results.
- Applying well established standards means that the research can be replicated, and then analysed and compared with similar studies.
- We can summarize vast sources of information and make comparisons across categories and over time; and,
- Personal bias can be avoided by keeping a 'distance' from participating subjects and using accepted computational techniques.
- Mathematical models help to derive precise and accurate results by analysing complex statistical data.
- It is useful in areas of planning and control where data is available in quantitative terms. Decisions are based on data and logic rather than intuition and judgment.
- Computer-based Statistical packages are available which facilitate analysis of qualitative data also (dummy variables are used to analyse the non-quantifiable data).

## Limitations of the Quantitative Approach to Management:

1. **Modelling Errors:** The quantitative approach is not immune to modelling errors. If the model is based on incorrect assumptions or flawed data, the resulting decisions may be inaccurate or misleading, potentially leading to suboptimal outcomes.
2. **Underestimation of the Human Element:** A limitation of the quantitative approach lies in its tendency to undervalue the crucial role of the human element in management. It may overlook the significance of intuition, experience, and contextual knowledge that managers possess, which can be valuable in decision-making processes.
3. **Lack of Realistic Perspective:** The mechanistic nature of the quantitative approach may detach it from the complexities of real-life managerial decision-making. Managers often face time constraints and incomplete data, making it impractical to wait for comprehensive information. This discrepancy between theory and reality can limit the approach's effectiveness.

4. **Unrealistic Assumptions:** The quantitative approach relies on assumptions that may not always hold. For instance, it assumes that all relevant variables are measurable and possess functional relationships. In practice, some variables may defy quantification, and the relationships between them may be intricate or uncertain.

5. **Challenges with Data Accuracy and Timeliness:** Practical challenges arise in maintaining up-to-date and accurate data, which can impact the effectiveness of the quantitative approach. Outdated or inaccurate data can undermine the reliability of models and subsequently influence decision outcomes.

## QUANTITATIVE ANALYSIS AND QUALITATIVE ANALYSIS

### ➢ *What is quantitative data?*

Qualitative and quantitative data differ in their approach and the type of data they collect. Quantitative data refers to any information that can be quantified — that is, numbers. If it can be counted or measured, and given a numerical value, it's quantitative in nature. Think of it as a measuring stick. Quantitative variables can tell you "How many," "how much," or "how often."

***Some examples of quantitative data:***

- How many people attended last week's webinar?
- How much revenue did our company make last year?
- How often does a customer rage click on this app?

To analyse these research questions and make sense of this quantitative data, you'd normally use a form of statistical analysis—collecting, evaluating, and presenting large amounts of data to discover patterns and trends. Quantitative data is conducive to this type of analysis because it's numeric and easier to analyse mathematically.

**Popular quantitative data collection methods are surveys, experiments, polls & more.*

### ➢ *What is qualitative data?*

Unlike quantitative data, qualitative data is descriptive, expressed in terms of language rather than numerical values. Qualitative data analysis describes information and cannot be measured or counted. It refers to the words or labels used to describe certain characteristics or traits.

You would turn to qualitative data to answer the "why?" or "how?" questions. It is often used to investigate open-ended studies, allowing participants (or customers) to show their true feelings and actions without guidance.

Some examples of qualitative data:

- Why do people prefer using one product over another?
- How do customers feel about their customer service experience?
- What do people think about a new feature in the app?

### More examples of quantitative and qualitative data

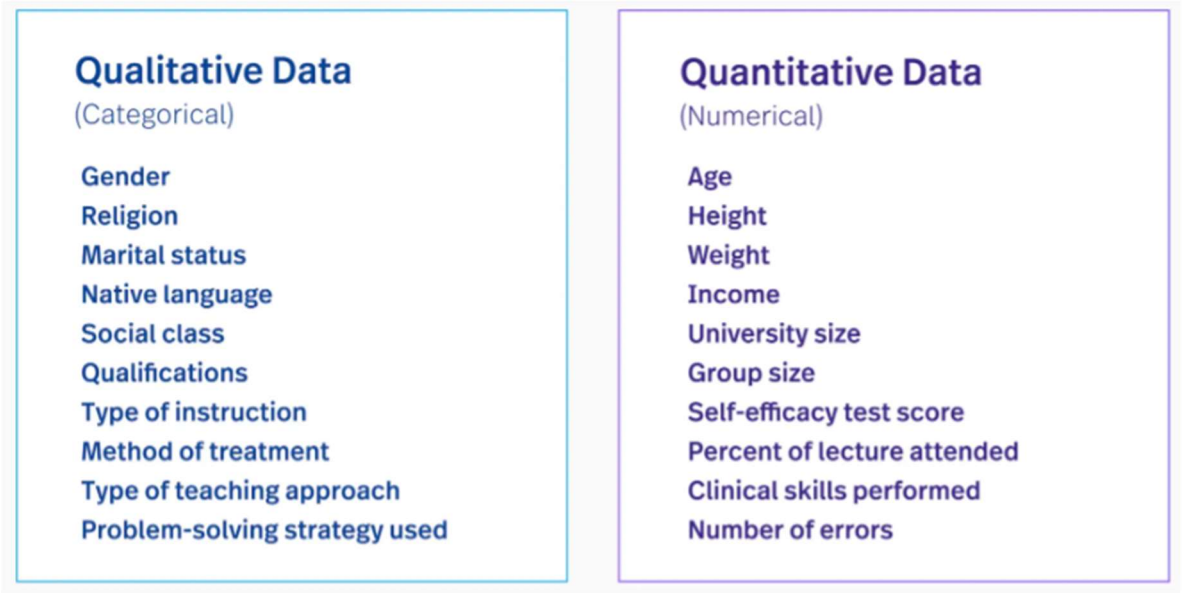
You've most likely run into quantitative and qualitative data today, alone. For the visual learner, here are some examples of both quantitative and qualitative data:

**Quantitative data example**

- The customer has clicked on the button 13 times.
- The engineer has resolved 34 support tickets today.
- The team has completed 7 upgrades this month.
- 14 cartons of eggs were purchased this month.

**Qualitative data example**

- My manager has curly brown hair and blue eyes.
- My coworker is funny, loud, and a good listener.
- The customer has a very friendly face and a contagious laugh.
- The eggs were delicious.




### ❖ Advantages of quantitative data

- It's relatively quick and easy to collect and it's easier to draw conclusions from.
- When you collect quantitative data, the type of results will tell you which statistical tests are appropriate to use.
- As a result, interpreting your data and presenting those findings is straightforward and less open to error and subjectivity.
  Another advantage is that you can replicate it. Replicating a study is possible because your data collection is measurable and tangible for further applications.

### Disadvantages of quantitative data

- Quantitative data doesn't always tell you the full story (no matter what the perspective).
- With choppy information, it can be inconclusive.
- Quantitative research can be limited, which can lead to overlooking broader themes and relationships.
- By focusing solely on numbers, there is a risk of missing larger focus information that can be beneficial.

### ❖ Advantages of qualitative data

- Qualitative data offers rich, in-depth insights and allows you to explore context.
- It's great for exploratory purposes.
- Qualitative research delivers a predictive element for continuous data.

### Disadvantages of qualitative data

- It's not a statistically representative form of data collection because it relies upon the experience of the host (who can lose data).
- It can also require multiple data sessions, which can lead to misleading conclusions.

## Quantitative analysis and Qualitative analysis

**The main difference between qualitative and quantitative analysis is that qualitative analysis uses non-numerical data, while quantitative analysis uses numerical data:**

## Quantitative analysis

- This type of analysis uses numerical data that can be easily converted into numbers without losing meaning. It's often used to determine the amount or concentration of a particular element or compound in a sample. It's also used to quantify variables and analyse statistical relationships. Quantitative analysis is a good way to confirm or test a theory or hypothesis.
- Quantitative data is collected through methods like surveys and experiments and analysed statistically to identify patterns.
- Quantitative analysis is generally concerned with measurable quantities such as weight, length, temperature, speed, width, and many more. The data can be expressed in a tabular form or any diagrammatic representation using graphs or charts. Quantitative data can be classified as continuous or discrete, and it is often obtained using surveys, observations, experiments or interviews.

## Qualitative analysis

- Qualitative analysis is concerned with the analysis of data that cannot be quantified. This type of data is about the understanding and insights into the properties and attributes of objects (participants). Qualitative analysis can get a deeper understanding of "why" a certain phenomenon occurs. The analysis can be used in conjunction with quantitative analysis or precede it.
- Qualitative data is gathered through interviews or observations and analysed by categorizing information to understand themes and insights.
- The typical data analysed qualitatively include colour, gender, nationality, taste, appearance, and many more as long as the data cannot be computed. Such data is obtained using interviews or observations.

# Quantitative Analysis Vs Qualitative Analysis

| Characteristics | Quantitative Analysis | Qualitative Analysis |
|---|---|---|
| **Definition** | It is an objective analysis that quantifies data. | It is a subjective analysis that is more concerned with non-statistical data that cannot be computed. |
| **Causative agents** | Typical data include measurable quantities such as length, size, weight, mass and many more. | Typical data include colour, gender, nationality, religion and many more. |
| **How is data collected?** | Close-ended questions with multiple-choice format, surveys, polls or questionnaires. | Open-ended questions with interviews and observations. |
| **How is data analysed?** | Mathematical and statistical analysis communicated with numbers, graphs and charts. | Verbal communication and analysis of summarizations, categorizations and interpretations. |
| **Advantages** | Impartiality, fast and reliable data collection methodology, larger sample sizes. | More detailed insights, methodology encourages deeper discussion. |
| **Disadvantages** | Unable to learn more context in answers, abnormal research environment, limited answers for data collection and insights. | Smaller sample sizes, more risk of biasness, requires highly skilled moderator. |
| **Common industries** | Finance, accounting, consulting. | Healthcare, health sciences, social sciences, legal, e-commerce, marketing. |

# OPERATIONS RESEARCH

Operations research (OR) is an analytical method of problem-solving and decision-making that is useful in the management of organizations. In operations research, problems are broken down into basic components and then solved in defined steps by mathematical analysis.

The subject of operations research was born during Second World War in United Kingdom and was used for military strategy. During World War II, a group of scientists, having representatives from mathematics, statistics, physical and social sciences were entrusted to the study of various military operations. This team was very successful and greatly contributed to the meticulous handling of entire operation and related problems of the operation. After the World War II, it was started applying in the fields of industry, trade, agriculture, planning and various other fields of economy.

## ❖ Various Definitions of OPERATIONS RESEARCH:

**"Operations Research is the art of winning wars without actually fighting."**
**– Aurther Clarke.**

**"Operations Research is defined as Scientific method for providing executive departments a quantitative basis for decisions regarding the operations under their control."**
**– P.M. Morse and G.E. Kimball.**

**"Operations Research is the application of scientific methods, techniques and tools to operation of a system with optimum solution to the problem."**
**– Churchman**

**"Operations Research is applied decision theory. It uses any scientific, mathematical or logical means to attempt to cope with problems that confront the executive, when he tries to achieve a thorough going rationally in dealing with his decision problem."**
**– D.W. Miller and M.K. Starr.**

**The process of operations research can be broadly broken down into the following steps:**

1. Identifying a problem that needs to be solved.
2. Constructing a model around the problem that resembles the real world and variables.
3. Using the model to derive solutions to the problem.
4. Testing each solution on the model and analysing its success.
5. Implementing the solution to the actual problem.

## ❖ Characteristics (Components) of operations research

There are three primary characteristics of all operations research efforts:

1. **Optimization**- The purpose of operations research is to achieve the best performance under the given circumstances. Optimization also involves comparing and narrowing down potential options.

2. **Simulation**- This involves building models or replications in order to try out and test solutions before applying them.

3. **Probability and statistics**- This includes using mathematical algorithms and data to uncover helpful insights and risks, make reliable predictions and test possible solutions.

## ❖ Main Features of Operations Research:

1. **Quantitative Analysis**: OR involves the use of mathematical models and quantitative techniques to analyse problems and derive optimal solutions.

2. **Operation Research both as an Art and a Science:** Scientific orientation of Operation Research is due to the inherent approach and scientific techniques which it uses to solve its problems. But it is only Art aspect which is used for implementing the decision for the entire company.

3. **Optimization**: It focuses on optimizing or maximizing/minimizing some objective function, subject to constraints, in order to find the best possible solution.

4. **Decision Support**: OR provides decision-makers with insights, recommendations, and tools to support informed decision-making in complex and uncertain environments.

5. **Multidisciplinary Approach**: OR integrates concepts and methodologies from various disciplines such as mathematics, economics, computer science, and engineering.

6. **Problem-solving Methodology**: It follows a systematic approach to problem-solving, which typically involves problem formulation, model development, solution algorithms, implementation, and evaluation.

7. **Utilisation of Information Technology (IT):** Help of Information Technology is taken extensively by Operation Research to solve complicated mathematical problems. This is because decision making approach of Operation Research is directly dependent on the extensive operation of computers.

8. **Application Orientation**: OR is driven by practical applications and real-world problems, aiming to provide actionable solutions to improve organizational performance and decision-making.

9. **Iterative Process**: OR often involves iterative processes of modelling, analysis, and decision-making, where solutions are refined based on feedback and changing conditions.

## ❖ Applications of Operations Research

Today, almost all fields of business and government utilizing the benefits of Operations Research. There are voluminous of applications of Operations Research. Although it is not feasible to cover all applications of O.R. in brief.

The following are the abbreviated set of typical operations research applications to show how widely these techniques are used today:

**1.Accounting**: Assigning audit teams effectively Credit policy analysis Cash flow planning Developing standard costs Establishing costs for byproducts Planning of delinquent account strategy Construction:

**2.Project scheduling, monitoring and control Determination of proper work force Deployment of work force Allocation of resources to projects Facilities Planning**: Factory location and size decision Estimation of number of facilities required Hospital planning International logistic system design Transportation loading and unloading Warehouse location decision.

**3. Finance**: Building cash management models Allocating capital among various alternatives Building financial planning models Investment analysis Portfolio analysis Dividend policy making

**4.Manufacturing:** Inventory control Marketing balance projection Production scheduling Production smoothing Marketing: Advertising budget allocation Product introduction timing Selection of Product mix Deciding most effective packaging alternative

**5.Organizational Behaviour / Human Resources:** Personnel planning Recruitment of employees Skill balancing Training program scheduling Designing organizational structure more effectively.

**6.Purchasing:** Optimal buying Optimal reordering Materials transfer

**7.Research and Development:** R & D Projects control R & D Budget allocation Planning of Product introduction

## ❖ Limitations of Operations Research

Operations Research has number of applications; similarly, it also has certain limitations. These limitations are mostly related to the model building and money and time factors problems involved in its application. Some of them are as given below:

**i) Distance between O.R. specialist and Manager**: Operations Researchers job needs a mathematician or statistician, who might not be aware of the business problems. Similarly, a manager is unable to understand the complex nature of Operations Research. Thus, there is a big gap between the two personnel.

**ii) Magnitude of Calculations:** The aim of the O.R. is to find out optimal solution taking into consideration all the factors. In this modern world these factors are enormous and expressing them in quantitative model and establishing relationships among these require voluminous calculations, which can be handled only by machines.

**iii) Money and Time Costs**: The basic data are subjected to frequent changes, incorporating these changes into the operations research models is very expensive. However, a fairly good solution at present may be more desirable than a perfect operations research solution available in future or after some time.

**iv) Non-quantifiable Factors:** When all the factors related to a problem can be quantifiable only then operations research provides solution otherwise not. The non-quantifiable factors are not incorporated in O.R. models. Importantly O.R. models do not take into account emotional factors or qualitative factors.

**v) Implementation:** Once the decision has been taken it should be implemented. The implementation of decisions is a delicate task. This task must take into account the complexities of human relations and behaviour and in some times only the psychological factors.

# DECISION THEORY

## What is Decision Theory?

Decision theory, also known as the theory of choice, is a branch of statistics that deals with making decisions based on the probability of the occurrence of an event. The chief purpose of decision theory is to identify the best course of action for the company with maximum payoffs.

Decision theory (DT) represents a generalized approach to decision making. It enables the decision maker:

- To analyse a set of complex situations with many alternatives and many different possible consequences
- To identify a course of action consistent with the basic economic and psychological desires of the decision maker

## ❖ Components (Elements) of Decision Theory

The Decision Theory consists of the following components:

1. **Decision Maker**

   The decision-maker is the person entitled to make the decisions on behalf of the company. The decision theory starts with the decision-maker when the identification of all the possibilities is done and subsequently, the outcomes such as profits, costs, and margins are calculated. After evaluation and identification, optimum decisions are taken, which maximize the productivity of the business. Optimum decisions involve maximizing profit, revenue, and margins and minimizing expenses such as cost, inefficiencies, and time.

2. **The course of action**

   A course of action comprises a series of steps and acts taken, which lead to an outcome. These acts are evaluated in decision theory and the decision-maker analyses the best course of action to achieve the goals of reaching optimal decisions.

3. **Outcomes**

   Outcomes are the result of the actions for each decision. These are the results that will be achieved when a particular course of action is taken into consideration.

4. **Payoffs**

   Payoffs are the returns on the outcome for a given decision. For instance, while selecting and evaluating a supplier, if X Inc. is providing the raw material at $8 per unit and Y Inc. provides the same at $8.5, choosing X Inc. over Y Inc. will save 50 cents on every unit purchased. The saving of 50 cents per purchase of each unit is considered to be the payoff.

## *Solution steps to any decision problem*:

The decision-making process consists of the following stages

1. **Identify the problem**

   To make a decision, you must first identify the problem you need to solve or the question you need to answer. Clearly define your decision.

2. Specify objectives and the decision criteria for choosing a solution.
3. Develop alternatives
4. Analyse and compare alternatives
5. Select the best alternative
6. Implement the chosen alternative
7. Verify that desired results are achieved

### ❖ Elements related to all decisions

- **Goals to be achieved**: objectives which the decision maker wants to achieve by his actions.
- **The decision maker:** Refers to an individual or an organization
- **Courses of action**: also called "Action" or "Decision Alternatives". They are under the control of decision maker
- **States of nature:** Exhaustive list of possible future events. Decision maker has no direct control over the occurrence of particular event.
- The preference or value system: criteria that the decision maker uses in making a choice of the best course of action
- **Payoff:** effectiveness associated with specified combination of a course of action and state of nature. Also known as profits or conditional values
- **Payoff table:** a summary table of the payoffs.
- Opportunity loss table: incurred due to failure of not adopting most favourable course of action or strategy. Found separately for each state of nature

## ❖ ***Decision making Environments:***

1. Decision making under certainty
   - Future "states of nature" are known
   - Will choose the alternative that has the highest payoff (or the smallest loss)
2. Decision making under uncertainty
   - Future "states of nature" are uncertain
   - Depends on the degree of decision maker's optimism
3. Decision making under risk

### 1] Decision making under Certainty

Such type of environment is very sure and certain by its nature. This means that all the information is available and at hand. Such data is also easy to attain and not very expensive to gather.

So, the manager has all the information he may need to make an informed and well thought out decision. All the alternatives and their outcomes can also be analysed and then the manager chooses the best alternative.

### 2] Decision making under Uncertainty

In the decision-making environment of uncertainty, the information available to the manager is incomplete, insufficient and often unreliable.

In an uncertain environment, everything is in a state of flux. Several external and random forces mean that the environment is most unpredictable.

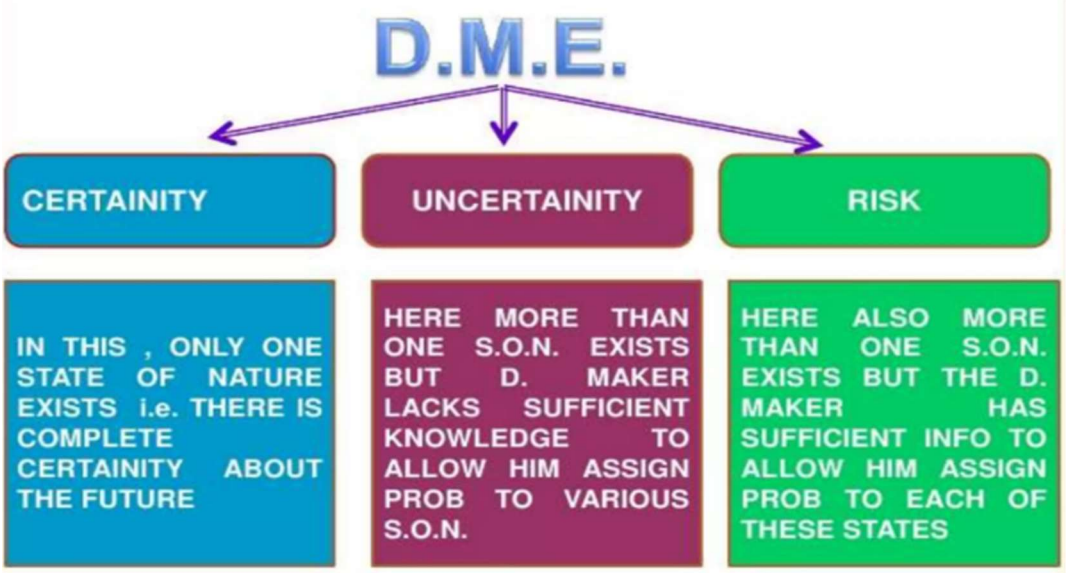
In these times of confusion (chaos), all the variables change fast. But the manager has to make sense of this mayhem (confusion) to the best of his ability. He must create some order, obtain some reliable data and make the best decision as per his judgment.

### 3] Decision making under Risk

Under the condition of risk, there is the possibility of more than one event taking place. Which means the manager has to first ascertain the possibility and probability of the occurrence or non-occurrence of the event.

The manager will generally rely on past experiences to make this deduction.

In this scenario too, the manager has some information available to him. But the availability and the reliability of the information is not guaranteed. He has to chart a few alternative courses of actions from the data he has.




## ➢ Criteria of Decision-Making under uncertainty

In the absence of knowledge about the probability of any state of nature (future) occurring, the decision-maker must arrive at a decision only on the actual conditional payoff values, together with a policy (attitude). There are several different criteria of decision-making in this situation. The criteria that we will discuss in this section include:

*(i) Criterion of Optimism (Maximax or Minimin)*
*(ii) Criterion of Pessimism (Maximin or Minimax)*
*(iii) Equally Likely Decision (Laplace) criterion*
*(iv) Criterion of Realism (Hurwicz Criterion)*
*(v) Criterion of Regret*

### (i) Criterion of Optimism (Maximax or Minimin)

In this criterion the decision-maker ensures that he should not miss the opportunity to achieve the largest possible profit (Maximax) or lowest possible cost (Minimin). Thus, he selects the alternative (decision choice or course of action) that represents the maximum of the maxima (or minimum of the minima) payoffs (consequences or outcomes). The working method is summarized as follows:

**(a)** Locate the maximum (or Minimum) payoff values corresponding to each alternative. (or course of action), then

**(b)** Select an alternative with the best anticipated payoff value (maximum for profit and minimum for cost).

Since in this criterion the decision-maker selects an alternative with the largest (or lowest) possible payoff values, it is also called an *Optimistic decision criterion*.

### (ii) Criterion of Pessimism (Maximin or Minimax)

In this criterion the decision-maker ensures that he would earn no less (or pay no more) than some specified amount. Thus, he selects the alternative that represents the maximum of the minima (or minimum of the maxima in case of loss) payoff in case of profits. The working method is summarized as follows:

**(a)** Locate the Minimum (or maximum in case of profit) payoff values in case of loss (or cost) data corresponding to each alternative, then

**(b)** Select an alternative with the best anticipated payoff value (maximum for profit and minimum for loss or cost).

Since in this criterion the decision-maker is conservative about the future and always anticipates the worst possible outcome (minimum for profit and maximum for cost or loss), it is called a *Pessimistic decision criterion*. This criterion is also known as *Wald's* criterion.

### (iii) Equally likely Decision (Laplace) Criterion

Since the probabilities of the stats of nature are not known, it is assumed that all states of nature will occur with equal probability, that is, each state of nature is assigned an equal probability. As states of nature are mutually exclusive and collectively exhaustive, the probability of each of these must be 1/ ***(number of states of nature).*** The working method is summarized as follows:

**(a)** Assign equal probability value to each state of nature by using the formula:

1/ (Number of states of nature)

**(b)** Compute the expected (or average) payoff for each alternative (course of action) by adding all the payoffs and dividing by the number of possible states of nature or by applying the formula:

*(Probability of state of nature j) x (Payoff value for the combination of alternative i and state of nature j)*

**(c)** Select the best expected payoff value (maximum for profit and minimum for cost).

This criterion is also known as the criterion of insufficient reason because, except in a few cases, some information of the likelihood of occurrence of the states of nature is available.

### (iv) Criterion of Realism (Hurwicz Criterion)

This criterion suggests that a rational decision-maker should be neither completely optimistic nor pessimistic and, therefore, must display a mixture of both. Hurwicz, who suggested this criterion, introduced the idea of a coefficient of optimism (denoted by α) to measure the decision-maker's degree of optimism. This coefficient lies between 0 and 1, where '**0**' represents a completer pessimistic attitude about the future and '**1**' represents a compete optimistic attitude about the future. Thus, if α is the coefficient of optimism, then (1- α) will represent the coefficient of pessimism.

The Hurwicz approach suggests that the decision maker must select an alternative that maximizes:

***H (Criterion of realism) = α (maximum in column) + (1- α) (Minimum in column)***

The working method is summarized as follows:

**(a)** Decide the coefficient of optimism α and then the coefficient of pessimism (1- α).

**(b)** For each alternative select the largest and smallest payoff values and multiply these with α and (1- α) values, respectively. Then calculate the weighted average, H by using the above formula.

**(c)** Select an alternative with the best anticipated weighted average payoff value.

### (v) Criterion of Regret

This criterion is also known as the opportunity loss decision criterion or minimax regret decision criterion because the decision-maker feels regret after adopting a wrong course of action (or alternative) resulting in an opportunity loss of payoff. Thus, he always intends to minimize this regret. The working method is summarized as follows:

**(a)** From the given payoff matrix develop an opportunity-loss (or regret) matrix as follows

(i) Find the best payoff corresponding to each state of nature, and

(ii) Subtract all other entries (payoff values) in that row from this value.

**(b)** For each course of action (Strategy or alternative) identify the worst or maximum regret value. Record this number in a new row.

**(c)** Select the course of action (alternative) with the smallest anticipated opportunity loss value.

## *Example 1:*

A food product's company is contemplating the introduction of a revolutionary new product with new packaging or replacing the existing product at much higher price (S**1**). It may even make a moderate change in the composition of the existing product, with a new packaging at a small increase in price (S**2**), or a small change in the composition of the existing product, except the word 'New' and a negligible increase in price (S**3**). The possible states of nature or events are: (i)high increase in sales (N**1**), (ii)no change in sales (N**2**), and (iii)decrease in sales (N**3**). The marketing department of the company worked out the payoffs in terms of yearly net profits for each of the strategies of three events (expected sale). This is represented in the following table:

| Stratagies | States of Nature | | |
|---|---|---|---|
| | N1 | N2 | N3 |
| S1 | 7,00,000 | 3,00,000 | 1,50,000 |
| S2 | 5,00,000 | 4,50,000 | 0 |
| S3 | 3,00,000 | 3,00,000 | 3,00,000 |

Which strategy should be the concerned executive choose on the basis of

**a) Maximin Criterion b) Maximax Criterion c) Minimax regret Criterion d) Laplace Criterion?**

## Solution:

The payoff matrix is rewritten as follows

| States of Nature | Stratagies | | |
|---|---|---|---|
| | S1 | S2 | S3 |
| N1 | 7,00,000 | 5,00,000 | 3,00,000 |
| N2 | 3,00,000 | 4,50,000 | 3,00,000 |
| N3 | 1,50,000 | 0 | 3,00,000 |

**a) <u>Maximin Criterion</u>**

**1. Select the minimum value from each strategy (column)**

**2. Select the Maximum value among the Minimum values**

| States of Nature | Stratagies | | | |
|---|---|---|---|---|
| | S1 | S2 | S3 | |
| N1 | 7,00,000 | 5,00,000 | 3,00,000 | |
| N2 | 3,00,000 | 4,50,000 | 3,00,000 | |
| N3 | 1,50,000 | 0 | 3,00,000 | **Maximum** |
| **Column Minimum** | **1,50,000** | **0** | **3,00,000** | **3,00,000** |

The maximum value of the column minimum is 3,00,000.

Hence, the company should adopt strategy **S3**

**b) Maximax Criterion**

1. **Select the maximum value from each strategy (column)**
2. **Select the Maximum value among the Minimum values**

| States of Nature | Stratagies | | | |
|---|---|---|---|---|
| | **S1** | **S2** | **S3** | |
| **N1** | 7,00,000 | 5,00,000 | 3,00,000 | |
| **N2** | 3,00,000 | 4,50,000 | 3,00,000 | |
| **N3** | 1,50,000 | 0 | 3,00,000 | **Maximum** |
| **Column Maximum** | **7,00,000** | **4,50,000** | **3,00,000** | **7,00,000** |

The maximum value of the column maximum is 7,00,000.

Hence, the company should adopt strategy **S1**

**c) Minimax Regret Criterion**

i) Select the maximum value from each States of Nature (Row)

| States of Nature | Stratagies | | | Maximum |
|---|---|---|---|---|
| | **S1** | **S2** | **S3** | |
| **N1** | 7,00,000 | 5,00,000 | 3,00,000 | **7,00,000** |
| **N2** | 3,00,000 | 4,50,000 | 3,00,000 | **4,50,000** |
| **N3** | 1,50,000 | 0 | 3,00,000 | **3,00,000** |

ii) Subtract the values of each state of nature (Rows) from the corresponding Row Maximum

iii) Identify the maximum value in each column of strategy.

| States of Nature | Stratagies | | |
|---|---|---|---|
| | **S1** | **S2** | **S3** |
| **N1** | 7,00,000-7,00,000 **=0** | 7,00,000-5,00,000 **= 2,00000** | 7,00,000-3,00,000 **= 4,00,000** |
| **N2** | 4,50,000 -3,00,000 **=1,50,000** | 4,50,000-4,50,000 **= 0** | 4,50,000-3,00,000 **=1,50,000** |
| **N3** | 3,00,000-1,50,000 **=1,50,000** | 3,00,000 - 0 **=3,00,000** | 3,00,000-3,00,000 **= 0** |
| **Column Maximum** | **1,50,000** | **3,00,000** | **4,00,000** |

iv) Among the column maximum values select the minimum value (Minimax regret)

v) In this example the Minimum of column Maximum is noticed through the **strategy S1**

vi) Hence the company should adopt the **minimum opportunity loss strategy S1.**

### d) Laplace Criterion

Since we do not know the probabilities of the states of nature, assume that they are equal. In this example, we would assume that each state of nature has a one third probability of occurrence. Thus,

| Strategy | Expected Return (Rs) |
|---|---|
| S1 | (7,00,000 + 3,00,000 + 1,50,000) / 3 = **3,83,333.33** |
| S2 | (5,00,000 + 4,50,000 + 0) /3 = **3,16,666.66** |
| S3 | (3,00,000 + 3,00,000 + 3,00,000) / 3 = **3,00,000** |

Since the largest expected return is from Strategy S1, the executive must select strategy S1.

## Example 2:

The following matrix gives the payoff (in Rs.) of different strategies (alternatives) S1, S2, and S3 against conditions (events) N1, N2, N3 and N4.

| Strategy | States of Nature | | | |
|---|---|---|---|---|
| | N1 | N2 | N3 | N4 |
| S1 | 4,000 | -100 | 6,000 | 18,000 |
| S2 | 20,000 | 5,000 | 400 | 0 |
| S3 | 20,000 | 15,000 | -2,000 | 1000 |

Indicate the decision taken under the following approaches **(a)Pessimistic (b)Optimistic, (c)Equal Probability (d)Regret (e)Hurwicz criterion. The degree of optimism being 0.7**.

## Solution:

The payoff matrix is rewritten as follows

| States of Nature | Stratagies | | |
|---|---|---|---|
| | S1 | S2 | S3 |
| N1 | 4,000 | 20,000 | 20,000 |
| N2 | -100 | 5,000 | 15,000 |
| N3 | 6,000 | 400 | -2,000 |
| N4 | 18,000 | 0 | 1,000 |

### a) Maximin Criterion (Pessimistic)

**1. Select the minimum value from each strategy (column)**
**2. Select the Maximum value among the Minimum values**

| States of Nature | Stratagies | | | |
|---|---|---|---|---|
| | S1 | S2 | S3 | |
| N1 | 4,000 | 20,000 | 20,000 | |
| N2 | -100 | 5,000 | 15,000 | |
| N3 | 6,000 | 400 | -2,000 | |
| N4 | 18,000 | 0 | 1,000 | **Maximum** |
| **Minimum** | **-100** | **0** | **-2,000** | **0** |

The maximum value of the column minimum is 0.

Hence, the company should adopt strategy **S2.**

## b) Maximax Criterion (Optimistic)

**1. Select the maximum value from each strategy (column)**
**2. Select the Maximum value among the Minimum values**

| States of Nature | Stratagies | | | |
|---|---|---|---|---|
| | S1 | S2 | S3 | |
| N1 | 4,000 | 20,000 | 20,000 | |
| N2 | -100 | 5,000 | 15,000 | |
| N3 | 6,000 | 400 | -2,000 | |
| N4 | 18,000 | 0 | 1,000 | **Maximum** |
| **Maximum** | **18,000** | **20,000** | **20,000** | **20,000** |

The maximum value of the column maximum is 20,000.
Hence, the company should adopt strategy **S2 or S3**

## (c) Equal Probability (Laplace Criterion)

Since equal probabilities are assumed for each state of nature, In this example, we would assume that each state of nature has a one fourth (1/4) probability of occurrence. Thus

| Strategy | Expected Return | |
|---|---|---|
| S1 | (4,000 -100+6000 +18000)/4 | = 6,975 |
| S2 | (20,000+5,000+400 +0) /4 | = 6,350 |
| S3 | (20,000+ 15,000 -2000 +1000) /4 | = **8,500** |

Since the largest expected return is from Strategy S3, the executive must select strategy S3.

## (d) Minimax Regret Criterion

i) Select the maximum value from each States of Nature (Row)

| States of Nature | Stratagies | | | Maximum |
|---|---|---|---|---|
| | S1 | S2 | S3 | |
| N1 | 4,000 | 20,000 | 20,000 | **20,000** |
| N2 | -100 | 5,000 | 15,000 | **15,000** |
| N3 | 6,000 | 400 | -2,000 | **6,000** |
| N4 | 18,000 | 0 | 1,000 | **18,000** |

ii) Subtract the values of each state of nature (Rows) from the corresponding Row Maximum

iii) Identify the maximum value in each column of strategy.

| States of Nature | Stratagies | | | |
|---|---|---|---|---|
| | S1 | S2 | S3 | |
| N1 | 20,000-4,000 **= 16,000** | 20,000-20,000 **= 0** | 20,000-20,000 **= 0** | |
| N2 | 15,000- (-100) **=15,100** | 15,000- 5,000 **=10,000** | 15,000- 15,000 **= 0** | |
| N3 | 6,0000- 6,000 **=0** | 6,0000- 400 **=5,600** | 6,0000- (-2,000) **= 8,000** | |
| N4 | 18,000- 18,000 **= 0** | 18,0000- 0 **= 18,000** | 18,0000 - 1,000 **= 17,900** | **Minimum value** |
| **Maximum** | **16,000** | **18,000** | **17,900** | **16,000** |

iv) Among the column maximum values select the minimum value (Minimax regret)

v) In this example the Minimum of column Maximum is noticed through the **strategy S1**

vi) Hence the company should adopt the **minimum opportunity loss strategy S1.**

## (e)Hurwicz criterion

It is given that the degree of optimism is 0.7, the degree of pessimism will be 1-0.7=0.3. According to Hurwicz, select a course of action that optimizes (minimum cost or maximum profit) the payoff value

$$H = \alpha\ (Best\ payoff) + (1-\alpha)\ (Worst\ payoff)$$

$$H = \alpha\ (maximum\ in\ column) + (1-\alpha)\ (Minimum\ in\ column)$$

| States of Nature | Stratagies | | |
|---|---|---|---|
| | S1 | S2 | S3 |
| N1 | 4,000 | 20,000 | 20,000 |
| N2 | -100 | 5,000 | 15,000 |
| N3 | 6,000 | 400 | -2,000 |
| N4 | 18,000 | 0 | 1,000 |
| **BEST payoff** | **18,000** | **20,000** | **20,000** |
| **WORST payoff** | **-100** | **0** | **-2,000** |
| **H** | | | |

**FOR S1 H = (0.7 x 18,000)+ (0.3 x -100)= 12,600 - 30 = 12,570**

**FOR S2 H = (0.7 x 20,000)+ (0.3 x 0 ) = 14,000 +0 = 14,000**

**FOR S3 H = (0.7 x 20,000)+ (0.3 x -2,000) = 14,000 – 600 = 13,400**

Since the course of action **S2** has the maximum return 14,000. Hence the manufacturer should adopt **S2.**

# Practice Problems:

1. A farmer wants to decide which of the three crops he should plant on his 100-acre form. The profit from each is dependent on the rainfall during the growing season. The farmer has categorized the amount of rainfall as high, medium and low. His estimated profit for each is shown in the table below:

| Rainfall | Estimated conditional profit (Rs.) | | |
|---|---|---|---|
| | Crop A | Crop B | Crop C |
| High | 8,000 | 3,500 | 5,000 |
| Medium | 4,500 | 4,500 | 5,000 |
| Low | 2,000 | 5,000 | 4,000 |

If the famer wishes to plant only on crop, decide which should be his 'best crop' using 1) Maximax Criterion 2) Maximin Criterion 3) Laplace Criterioin,4) Minimax regret Criterion and 5) Hurwicz Criterion (farmer's degree of optimism being 0.6).

2. The research department of Hindustan Lever has recommended to the marketing department to launch a shampoo of three different types. The marketing manager has to decide one the of types of shampoo to the launched under the following estimated payoffs for various levels of sales:

| Types of shampoo | Estimated levels of sale (units) | | |
|---|---|---|---|
| | 15,000 | 10,000 | 5,000 |
| Egg Shampoo | 30 | 10 | 10 |
| Clinic Shampoo | 40 | 15 | 5 |
| Deluxe Shampoo | 55 | 20 | 5 |

What will be the marketing manager's decision based on 1) Maximax Criterion 2) Maximin Criterion 3) Laplace Criterioin,4) Minimax regret Criterion.

3. A super bazar must decide on the level of supplies it must stock to meet the needs of its customers during Diwali days. The exact number of customers is not known, but it is expected to be in one of the four categories, E1, E2, E3, and E4. Four levels of supplies are thus suggested with level j being ideal (from the viewpoint of incurred costs) if the number of customers falls in category j. Deviations from the ideal levels results in additional costs either because extra supplies are stocked needlessly or because demand cannot be satisfied. The table below provides these costs in thousands of rupees.

| Customer Category | Supplies level | | | |
|---|---|---|---|---|
| | A1 | A2 | A3 | A4 |
| E1 | 7 | 12 | 20 | 27 |
| E2 | 10 | 9 | 10 | 25 |
| E3 | 23 | 20 | 14 | 23 |
| E4 | 32 | 24 | 21 | 17 |

Determine the best level of inventory is specified by the supply level using 1) Maximax Criterion 2) Maximin Criterion 3) Laplace Criterioin,4) Minimax regret Criterion.

## ➢ Criteria of Decision making under risk

The most widely used criterion for evaluating the alternative courses of action, is the ***Expected Monetary Value (EMV)*** or Expected Utility. The objective of decision-making under this condition is to optimize the expected payoff.

- **The Expected Monetary Value (EMV):** The Expected Monetary Value (EMV) for the specified course of action is the weighted average payoff, i.e., the sum of product of the payoff for the several combinations of courses of action and states of nature multiplied by the probability of occurrence of each outcome.

In the case of probabilistic situation each course of action can lead to a number of different possible outcomes. So the decision-maker is always interested to know two things (i) the average payoff or expected value of an outcome (ii) risk involved in the particular course of action. Both of these given information's can be obtained by using the concept of expected value.

Let us assume that iti is possible to attach a measure of probability to each value assumed by the specified state of nature. Then the expected monetary value corresponding to each course of action is given by

$$EMV\ (S_i) = \sum_{j=1}^{n} P(O_j).a_{ij}$$

Where $S_i$= course of action.

$P(O_j)$ = probability of occurrence of state of nature $O_j$

**n**= number of possible states of nature.

$a_{ij}$= payoff associated with course of action i, and state of nature $O_j$

- **Expected Opportunity Loss (EOL):** An alternative approach to maximizing expected monetary value (EMV) is to minimize expected opportunity loss (EOL). Expected opportunity loss or expected value of regrets are calculated in the same manner as the expected payoffs in the EMV criterion. The opportunity loss can be obtained separately for each course of action by first obtaining the best state of nature for the prescribed course of action and then taking the

difference between that best outcome (or conditional profit) and each outcome for those courses of action. The opportunity loss for each course of action is known as the conditional opportunity loss (COL).

After calculating the opportunity loss value for each course of action, the expected opportunity loss (EOL) for i[th] course of action $S_i$ is then computed by

$$EOL\ (S_i) = \sum_{j=1}^{n} COL(S_i, O_j).P(O_j)$$

- **Expected Value of Perfect Information (EVPI)**

The expected profit with perfect information is the expected return, in the long run, if we have perfect information before a decision is made. The Expected value of Perfect Information (EVPI) may be defined as the maximum amount one would be willing to pay, to acquire perfect information as to which event would occur. EPPI represents the maximum obtainable EMV with perfect information as to which event will actually occur (as calculated before perfect information is received). If EMV* represents the maximum obtainable EMV without perfect information, perfect information would increase expected profit from EMV* up to the value of EPPI, so the amount of that increase would be equal to EVPI, thus we have

EVPI= EPPI - EMV*

# Decision Tree

• It is a decision support tool that uses a tree-like graph or model of decisions and their possible consequences, including chance event outcomes, resource costs, and utility

• Commonly used in operations research, specifically in decision analysis, to help identify a strategy most likely to reach a goal. Another use is as a descriptive means for calculating conditional probabilities

• It enables people to decompose a large complex decision problem into several smaller problems

**A decision tree consists of 3 types of nodes**: -

1. Decision nodes - commonly represented by squares.
2. Chance nodes - represented by circles.
3. End nodes - represented by triangles/ellipses.

A decision tree has only burst nodes (splitting paths) but no sink nodes (converging paths)

**Advantages of decision trees**:

• Are simple to understand and interpret
• Have value even with little hard data
• Possible scenarios can be added
• Worst, best and expected values can be determined for different scenarios
• Use a white box model. If a given result is provided by a model
• Can be combined with other decision techniques e.g. Net Present Value calculations

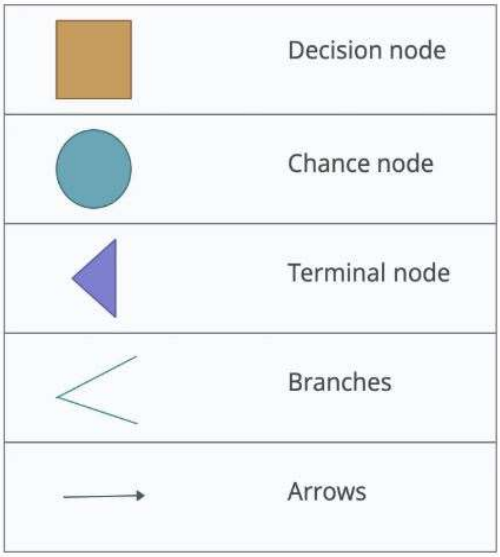
**Disadvantages of decision trees**:

• For data including categorical variables with different number of levels, information gain in decision trees is biased in favour of those attributes with more levels

• Calculations can get very complex particularly if many values are uncertain and/or if many outcomes are linked

## Decision Tree Symbols

Understanding the symbols used in a decision tree is essential for interpreting and creating decision trees effectively. Here are the main symbols and their meanings:




- **Decision node:** A point in the decision tree where a choice needs to be made. Represents decisions that split the tree into branches, each representing a possible choice.
- **Chance node:** A point where an outcome is uncertain. Represents different possible outcomes of an event, often associated with probabilities.
- **Terminal (or end) node:** The end point of a decision path. Represents the final outcome of a series of decisions and chance events, such as success or failure.
- **Branches:** Lines that connect nodes to show the flow from one decision or chance node to the next. Represent the different paths that can be taken based on decisions and chance events.
- **Arrows:** Indicate the direction of flow from one node to another. Show the progression from decisions and chance events to outcomes.

## Process or Steps of Decision Tree Analysis

Decision trees help finance experts make better decisions by allowing them to understand what they are stepping into before committing a huge sum of money or resources. To analyse uncertain outcomes and reach the most logical solution, you can create decision trees by using the following five steps:

### Step 1: *Start With One Idea*

Design a decision tree diagram with one main idea. To start building a tree, analysts should add a decision node first before adding single branches to the various decisions they are deciding between.

### Step 2: *Add Chance and Decision Nodes*

After including the primary idea in the tree, continue to add chance or decision nodes after each decision to expand the tree further. A chance node usually needs an alternative branch to be added after it because there could be more than one outcome for making that decision.

**Step 3:** ***Expand the Tree Till the End Points***

Keep adding chance and decision nodes to each branch of the decision tree until you reach endpoints and cannot expand the tree any further. At this point, add end nodes to your tree to highlight the completion of the tree creation process.

**Step 4:** ***Calculate tree values***

Ideally, your decision tree will have quantitative data associated with it. The most common data used in decision trees is monetary value.

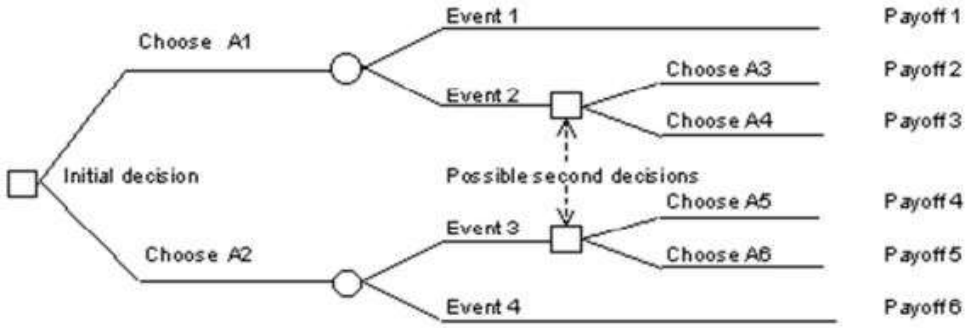
For example, it'll cost your company a specific amount of money to build or upgrade an app. It'll also cost more or less money to create one app over another. Writing these values in your tree under each decision can help you in the decision-making process.

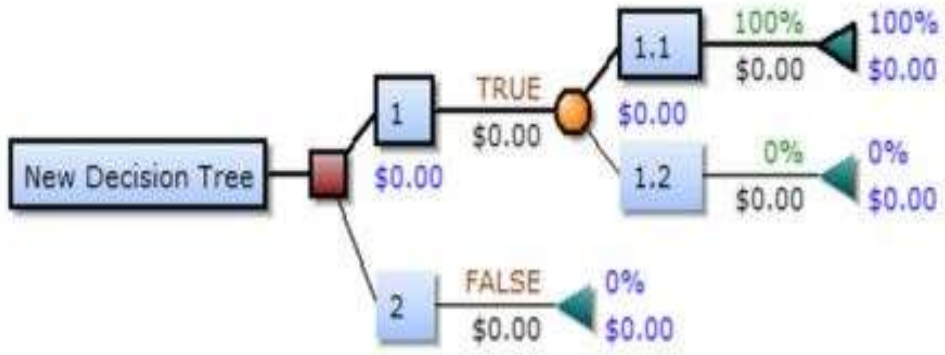
- Expected value (EV) = (First possible outcome x Likelihood of outcome) + (Second possible outcome **x** Likelihood of outcome) - Cost

**Step 5:** ***Evaluate outcomes***

Once you have your expected outcomes for each decision, determine which decision is best for you based on the amount of risk you're willing to take. The highest expected value may not always be the one you want to go for. That's because, even though it could result in a high reward, it also means taking on the highest level of project risk.

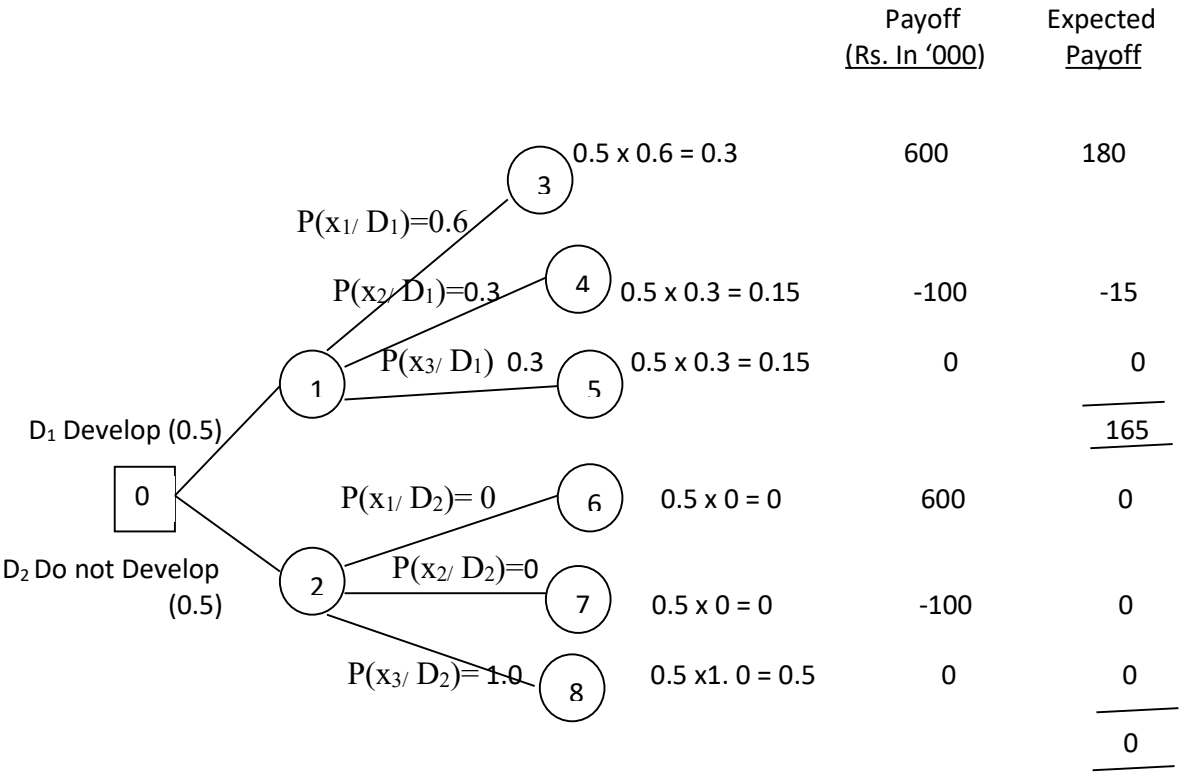
Examples of decision trees



**Example 1:** You are given the following estimates concerning a research and development programme:

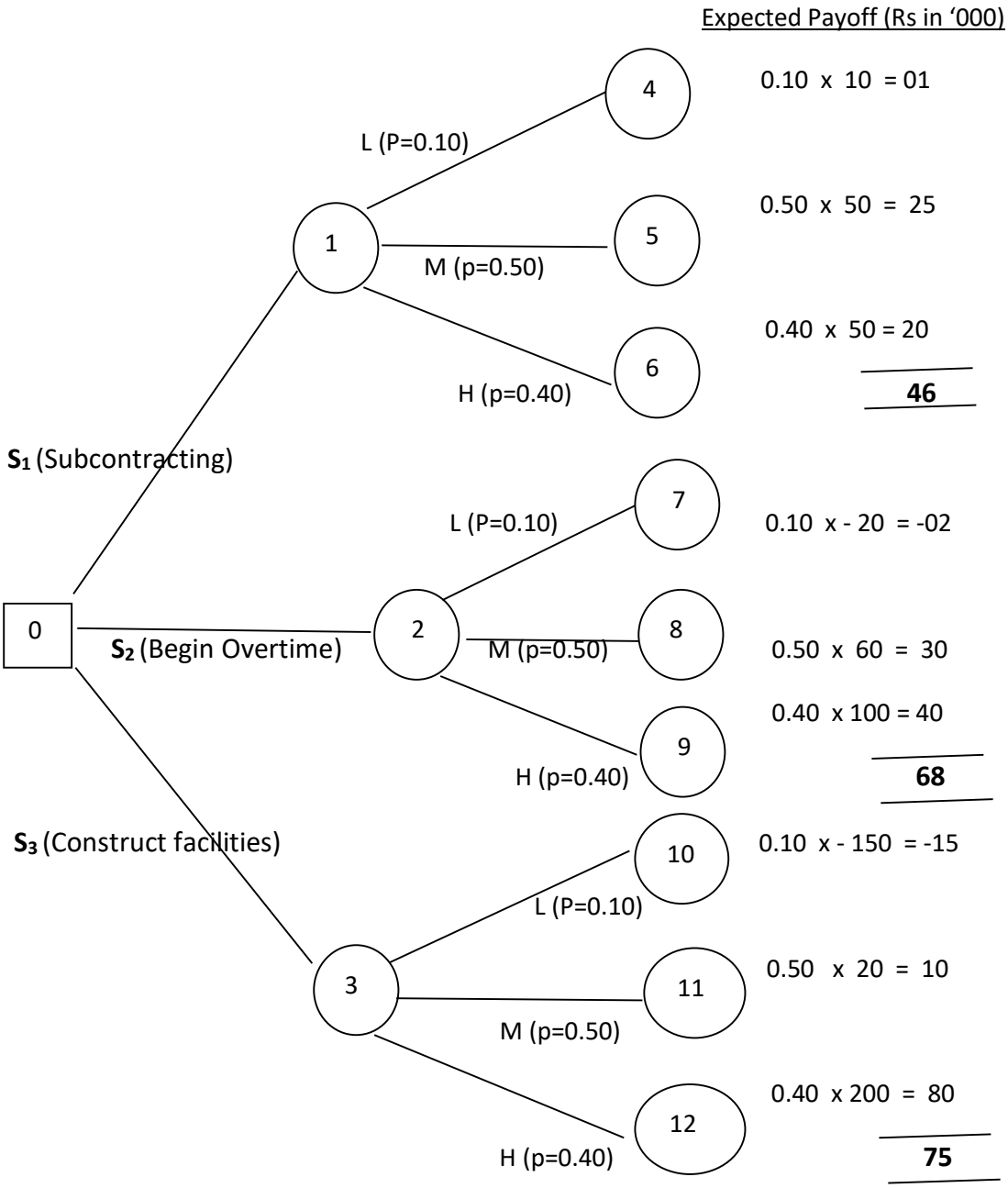
| Decision | Probability of Decision $D_i$ Given Research R $P(D_i / R)$ | Outcome Number | Probability of Outcome $x_i$ Given $D_i$ $P(x_i / D_i)$ | Payoff Value of Outcome Xi (Rs. In '000) |
|---|---|---|---|---|
| Develop | 0.5 | 1 | 0.6 | 600 |
| | | 2 | 0.3 | -100 |
| | | 3 | 0.1 | 0 |
| Do Not Develop | 0.5 | 1 | 0.0 | 600 |
| | | 2 | 0.0 | -100 |
| | | 3 | 1.0 | 0 |




**Example 2:** A glass factory specializing in crystal is developing a substantial backlog and the firm's management is considering three courses of action: Arrange for subcontracting ($S_1$), begin overtime production ($S_2$), and construct new facilities ($S_3$). The correct choice depends largely upon future demand which may be low, medium, or high. By consensus, management ranks the respective probabilities as 0.10, 0.50, and 0.40. A cost analysis reveals the effect upon the profits that is shown in the table below.

| Demand | Probability | Course of Action | | |
|---|---|---|---|---|
| | | $S_1$ (Subcontracting) | $S_2$ (Begin Overtime) | $S_3$ (Construct Facilities) |
| Low (L) | 0.10 | 10 | -20 | -150 |
| Medium (M) | 0.50 | 50 | 60 | 20 |
| High (H) | 0.40 | 50 | 100 | 200 |

Show this decision situation in the form of a decision tree and indicate the most preferred decision and the corresponding expected value.




Since node 3 has highest expected payoff, the decision at node 0 will be to choose the course of action $S_3$ that is, **construct new facilities.**

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